

# Non-Singlet spin structure function in the valon model and low x scaling behavior of $g_1^{NS}$ and $g_1^p$

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A next-to-leading order QCD calculation of non-singlet spin structure function,  $g_1^{NS}$  is presented within the valon representation of Hadrons. In the valon model, it is assumed that a nucleon is composed of three dressed valence quarks: the valons which have their own internal structure, the valence quark with its associated sea quarks and gluons. The results are in good agreement with all available data from SMC, E143, HERMES and with the newly released data from COMPASS experiments. It appears that the small x tail of  $g_1^{NS}$  can be described by a single Regge-type exchange. The relevant parameter of this exchange is given. Finally we show that the polarized proton structure function has a scaling behavior at small x. The relevant parameters of this behavior are given too.

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## I. INTRODUCTION

Deep Inelastic Scattering (DIS) of leptons from the nucleon has served as an important tool for the investigation of the nucleon substructure and is one of the key areas for testing

the Quantum ChromoDynamics (QCD).

Spin is a fundamental properties of the nucleon and the spin structure of nucleon has been the subject of heated debates over the past twenty years. The key question is that how the spin of the nucleon is distributed among its constituent partons. That is, the determination and understanding of the shape of quarks and gluon spin distribution functions,  $\delta q_f(x, Q^2)$ , have become an important issue.

We utilized the valon model [1] to study the polarized nucleon structure. In the valon model, it is assumed that a nucleon is composed of three dressed valence quarks: the valons. Each valon has its own internal structure, the valence quark with its associated sea quarks and gluons which can be probed at high enough  $Q^2$ . At low  $Q^2$ , a valon behaves as a valence quark. The valons play a role in scattering problems as the constituents do in bound state problems. It is assumed that the valons stand at a level in between hadrons and partons and that the valon distributions are independent of the probe or  $Q^2$ . In this representation a valon is viewed as a cluster of its own partons. The evolution of the parton distributions in a hadron is effected through the evolution of the valon structure, as the higher resolution of a probe reveals the parton content of the valon. This model has yielded excellent results for un-polarized structure functions, [1–6]. It has also been applied to the polarized nucleon structure function [7, 8] with interesting results .

In this paper we would like to concentrate on the non-singlet part of the polarized nucleon structure function; because of its simplicity and thus its transparency. In addition, there are more accurate data which are extended to fairly small  $x$  region:  $x < 0.01$ . That makes the comparison with the theory more meaningful. Recently COMPASS Collaboration released data on  $g_1^{NS}$  to test the Bjorken sum rule with more accuracy [9]. Therefore, in this paper we have further attempted to show the application of using this model for studying the nucleon structure functions.

This paper is organized as follows. In section 2, a brief general outline is presented on the calculation of the polarized nucleon structure function in the valon model. Then we calculate the non-singlet spin structure function in section 3. Finally, section 4 is devoted to study the Regge behavior of  $g_1^{NS}$  at small  $x$  and scaling behavior of  $g_1^p$ . Then, we will finish with conclusions.

## II. VALON MODEL AND POLARIZED HADRON STRUCTURE FUNCTION

The connection between bound state problem at the hadronic scale that occurs only at low  $Q^2$  and deep inelastic scattering at high  $Q^2$  can be investigated by introducing the valons. Each valon is a dressed valence quark, i.e., each being a valence quark with its associated sea quarks and gluons which can be resolved only at high  $Q^2$ . At low  $Q^2$  valon behaves as a CQ because its internal structure can not be resolved. Thus the valon distribution in a hadron is the wave-function square of the CQs, whose structure functions are described by PQCD at high  $Q^2$  [2].

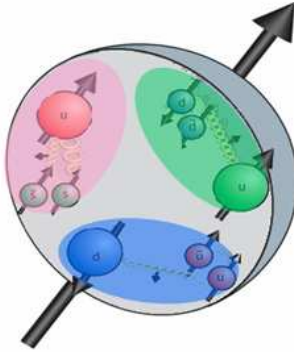


FIG. 1: (Color online) The schematic picture of the valon model.

The valon model is essentially a two components model. In this framework, the structure function  $F_2(x, Q^2)$  of a hadron is a convolution of the valon distribution  $G_{valon}^h(y)$  and the structure function of the valon,  $F_2^{valon}(z, Q^2)$

$$F_2^h(x, Q^2) = \sum_{valon} \int_x^1 dy G_{valon}^h(y) F_2^{valon}\left(\frac{x}{y}, Q^2\right) \quad (1)$$

In a similar way the parton distribution functions in a hadron can be obtained by

$$q(x, Q^2) = \sum_{valon} \int_x^1 dy G_{valon}^h(y) q^{valon}\left(\frac{x}{y}, Q^2\right) \quad (2)$$

where  $G_{valon}^h(y)$  is the valon distribution inside the hadron. It means the probability of finding a valon with momentum fraction of  $y$  in the hadron. The description of the  $G_{valon}^h(y)$  is given in [1, 2].

$valon(j)$	$N_j$	$\alpha_j$	$\beta_j$	$a_j$	$b_j$	$c_j$	$d_j$
$U$	3.44	0.33	3.58	-2.47	5.07	-1.859	2.780
$D$	-0.568	-0.374	4.142	-2.844	11.695	-10.096	14.47

TABLE I: Numerical values of the parameters in Eq. (4).

In polarized case, the helicity distributions of various partons in a hadron in the framework of this model are given by:

$$\delta q_i^h(x, Q^2) = \sum \int_x^1 \frac{dy}{y} \delta G_{valon}^h(y) \delta q_i^{valon}\left(\frac{x}{y}, Q^2\right) \quad (3)$$

where  $\delta G_{valon}^h(y)$  is the helicity distribution of the valon in the hosting hadron (probability of finding the polarized valon inside the polarized hadron). It is related to unpolarized valon distribution by:

$$\delta G_j(y) = \delta F_j(y) G_j(y) = N_j y^{\alpha_j} (1-y)^{\beta_j} (1 + a_j y^{0.5} + b_j y + c_j y^{1.5} + d_j y^2) \quad (4)$$

$G_j(y)$  are the unpolarized valon distributions, where  $j$  refers to U and D type valons (Regrettably, the above equation was erroneous in Ref. [7] in which  $\delta G_j(y)$  was replaced by  $\delta F_j(y)$ ). Polarized valon distributions are determined by a phenomenological argument for a number of hadrons [7, 8]. We summarized the parameters for Eq.(2.4) in Table 1.

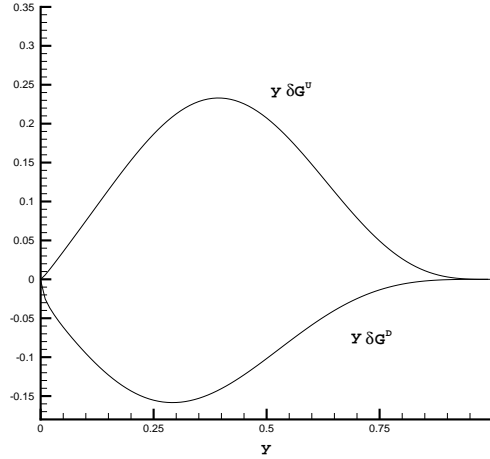


FIG. 2: Polarized valon distributions for U and D valons inside the proton.

We come back to Eq.(3),  $\delta q_i^{valon}(z = x/y, Q^2)$  is the polarized parton distribution in the valon . Polarized parton distributions inside the valon are evaluated according to the DGLAP evolution equation subject to physically sensible initial conditions.

$$\delta q^{NS\pm}(n, Q^2) = \{1 + \frac{\alpha_s(Q^2) - \alpha_s(Q_0^2)}{2\pi} (\frac{-2}{\beta_0}) (\delta \mathbf{P}_{NS\pm}^{(1)n} - \frac{\beta_1}{2\beta_0} \delta \mathbf{P}_{qq}^{(0)n})\} \mathbf{L}^{-(\frac{2}{\beta_0})\delta P_{qq}^{(0)n}} \delta q^{NS\pm}(n, Q_0^2) \quad (5)$$

$$\begin{pmatrix} \delta q^S(n, Q^2) \\ \delta g(n, Q^2) \end{pmatrix} = \{ \mathbf{L}^{-(\frac{2}{\beta_0})\delta \hat{P}^{(0)n}} + \frac{\alpha_s(Q^2)}{2\pi} \hat{\mathbf{U}} \mathbf{L}^{-(\frac{2}{\beta_0})\delta \hat{P}^{(0)n}} - \frac{\alpha_s(Q_0^2)}{2\pi} L^{-(\frac{2}{\beta_0})\delta \hat{P}^{(0)n}} \hat{\mathbf{U}} \} \begin{pmatrix} \delta q^S(n, Q_0^2) \\ \delta g(n, Q_0^2) \end{pmatrix} \quad (6)$$

The detail of above equations are given at [10]. As presented in [7] and [8], we have calculated the polarized nucleon structure function in the valon model. We have worked in  $\overline{MS}$  scheme with  $\Lambda_{QCD} = 0.22 \text{ GeV}$  and  $Q_0^2 = 0.283 \text{ GeV}^2$ . The initial motivation for this value of  $Q_0^2$  comes from the phenomenological consideration that requires us to choose the initial input densities as  $\delta(z-1)$  at  $Q_0^2$ . The valon structure function has the property that it becomes  $\delta(z-1)$  as  $Q^2$  is extrapolated to  $Q_0^2$  (beyond the region of validity). This mathematical boundary condition means that the internal structure of the valon can not be resolved at  $Q_0^2$  in the NLO approximation. It also means that at initial scale of  $Q_0^2$ , the nucleon can be considered as a bound state of three valence quarks that carry all the momentum and the spin of the nucleon. As  $Q^2$  is increased other partons can be resolved at the nucleon. It is also interesting to note that this value of  $Q^2$  is very close to the transition region reported by the CLAS Collaboration. Measurement of the first moment of the proton structure function at CLAS shows that there is a transition region around  $Q^2 = 0.3 \text{ GeV}^2$  [11] . Therefore the initial input densities to solve the DGLAP equations inside the valon are

$$\delta q^{NS}(z, Q_0^2) = \delta q^S(z, Q_0^2) = \delta(z-1) \quad (7)$$

$$\delta g(z, Q_0^2) = 0 \quad (8)$$

Thus their moments are

$$\delta q^{NS}(n, Q_0^2) = \delta q^S(n, Q_0^2) = \int_0^1 z^{n-1} \delta(z-1) dz = 1 \quad (9)$$

$$\delta g(n, Q_0^2) = 0 \quad (10)$$

In the valon model, the hadron structure is obtained by the convolution of valon structure and its distribution inside the hadron. Having specified the various components that contribute to the spin of a valon, we now turn to the polarized hadron structure, which is obtained by a convolution integral as follows:

$$g_1^h(x, Q^2) = \sum_{valon} \int_x^1 \frac{dy}{y} \delta G_{valon}^h(y) g_1^{valon}\left(\frac{x}{y}, Q^2\right). \quad (11)$$

The valon structure is generated by perturbative dressing in QCD. In such processes with massless quarks, helicity is conserved and therefore, the hard gluons can not induce sea quark polarization perturbatively. According to this description, it turns out that sea polarization is consistent with zero [7]. This finding is supported by HERMES experiment and by released data from COMPASS experiment [12–15]. Therefore we have no sea polarization in our model.

Using the initial conditions in Eq. (9) and Eq. (10), the calculation of the PPDFs inside the valon follow from the standard DGLAP evolution equations. The algorithm for calculation of the PPDFs inside the proton can be decomposed in the following three steps:

- Calculating PPDFs in the valon by using the DGLAP equations;
- With a phenomenological approach, one can find the helicity distributions for the valons, Figure 2. These functions are  $Q^2$ -independent. Since we find the valon helicity distributions, one can use them to calculate the polarized nucleon structure up to  $Q^2 = 10^7 GeV^2$ ;
- By using the convolution integral (equations (3) and (11)), one finds PPDFs in the nucleon and polarized structure function;

There is an excellent agreement between the model predictions with the experimental data for spin structure functions. A sample is given in Figure 3.

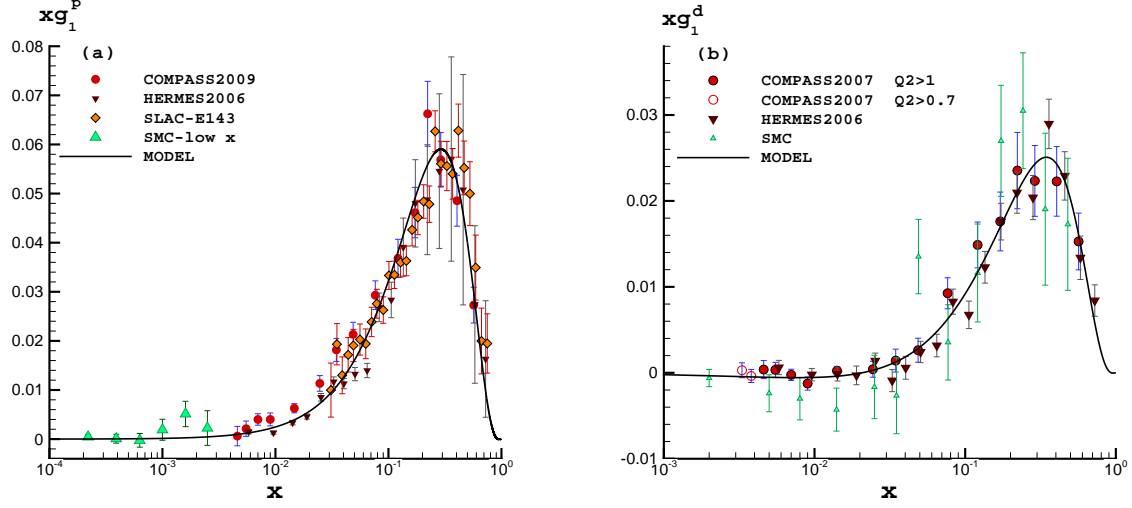


FIG. 3: (Color online) (a) Polarized proton structure function,  $xg_1^p$  at  $Q^2 = 5\text{GeV}^2$  ( $\chi^2/N = 1.7$ ). (b) Polarized deuteron structure function,  $xg_1^d$  at  $Q^2 = 3\text{GeV}^2$  ( $\chi^2/N = 1.33$ ). The results from model [7] are compared with the experimental data [9, 12, 14, 16–18]. The data from [9] are newly released data from COMPASS.

### III. NON-SINGLET SPIN STRUCTURE FUNCTION

The non-singlet polarized parton distribution function is defined as:  $\Delta q^{NS}(x, t) = (\Delta u - \Delta d)(x, t)$ ; with  $t = \ln \frac{Q^2}{\Lambda^2}$ .

The DGLAP equation for  $\Delta q^{NS}(x, t)$  is:

$$\frac{d}{dt} \Delta q^{NS} = \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) \Delta q^{NS}(y, t) \quad (12)$$

where  $P(z = x/y)$  is the NLO spin-dependent splitting function [10]. As mentioned before, in our calculations  $\Delta q_{sea} \simeq 0$ , therefore  $\Delta q^{NS}$  becomes  $(\Delta u_V - \Delta d_V)(x, t)$  which is shown in Figure 4 in comparison with other global fits. The non-singlet spin structure function is defined as:

$$g_1^{NS} \equiv g_1^p - g_1^n = 2[g_1^p - g_1^d/(1 - 1.5\omega_D)] \quad (13)$$

where  $\omega_D = 0.058$  accounts for the D-state admixture in the deuteron wave function.

The  $g_1^{NS}$  data is consistent with the quark model and the perturbative QCD predictions in the valence region  $x > 0.2$  [23].

In Figure 5(a), the  $x$  dependence of  $xg_1^{NS}$  is shown in comparison with data from HERMES, E143, SMC and also with the newly released data from COMPASS [9]. The re-

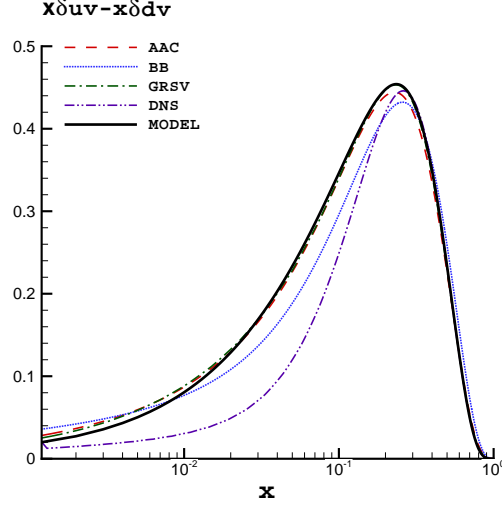


FIG. 4: (Color online) The non-singlet polarized parton distribution function in the valon model in comparison with other global fits [19–22].

sults are in very good agreement with experimental data for the entire measured range of  $x$ . In Figure 5(b) The evolution of the Bjorken integral, derived from the Figure 5(a),  $\int_{x_{min}}^1 (\frac{1}{x}(xg_1^{NS}))dx = \int_{x_{min}}^1 (g_1^{NS})dx$  as a function of  $x_{min}$  is shown for the model compared with the recent HERMES and COMPASS Collaboration data [9, 12]. Note that about 50 percent of the sum rule comes from  $x$  values below about 0.12 and that about 10-20 percent comes from values of  $x$  less than about 0.01. It shows that,  $g_1^{NS}$  receives a considerable contribution from the small  $x$  region. Thus, it seems that investigation of small  $x$  region of the structure function is important. In the following section we will consider  $g_1^{NS}$  at this region.

In Table 2 we compared the integral over different  $x$  ranges at different scales of  $Q^2$ , as determined from the valon model with the experimental results from COMPASS, HERMES, E143, E154, E155, SMC and JLAB also with the recent results from NN Collaborations [9, 11, 16–18, 24–27]. We have used data from JLAB for  $Q^2 > 0.5 GeV^2$  to make sure that the non perturbative effects are small .

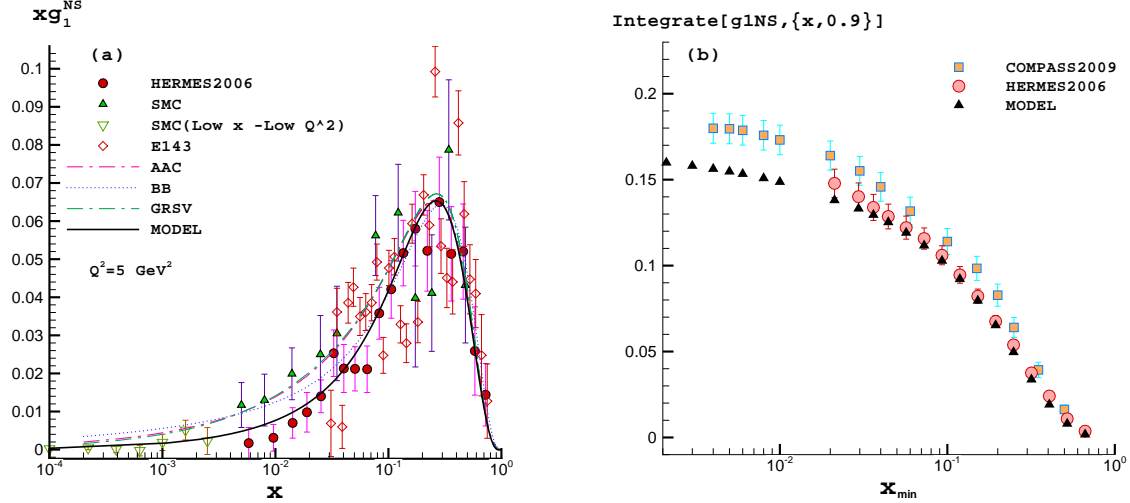


FIG. 5: (Color online) (a)  $xg_1^{NS}$  at  $Q^2 = 5\text{GeV}^2$  compared with the experimental data and the results from global fits [19–21] ( $\chi^2_N = 1.25$ ). (b) The integral of  $g_1^{NS}$  over the range  $0.02 < x < 0.9$  measured by HERMES and COMPASS Collaborations as a function of the low  $x$  limit of integration,  $x_{min}$ , evaluated at  $Q^2 = 10\text{GeV}^2$  in comparison with our results.

#### IV. REGGE BEHAVIOR OF $g_1^{NS}$ AND FULL $g_1^p$ AT SMALL $x$

In all the results from experimental data for unpolarized and polarized structure functions, it is seen that these structure functions increase when  $x$  decreases and  $Q^2$  increases for fixed values of  $x$  and  $Q^2$  respectively. The small  $x$  behavior of spin dependent structure functions reflects the high energy behavior of the polarized virtual compton scattering total cross section with increasing total CM energy squared  $W^2$  since  $W^2 = Q^2(\frac{1}{x} - 1)$ . When  $W^2 \gg Q^2$ ,  $x$  is small and  $W^2 \approx Q^2/x$  and then at this region the structure functions have scaling behavior. This is, by definition, the Regge limit and so the Regge pole exchange picture is therefor quite appropriate for the theoretical description of this behavior [28]. The small  $x$  or high energy behavior of the spin structure function is an important issue for the extrapolation of data needed to test spin sum rules for the first moment of  $g_1$ . The small  $x$  measurements, besides reducing the error on the first moment, would provide valuable information about Regge and QCD dynamics at small  $x$  where the shape of  $g_1$  is particularly sensitive to the different theoretical inputs.

In the case of unpolarized structure function,  $F_2$ , it is believed that a Regge trajectory combined with a soft and a hard pomeron can accurately represent the experimental data [29].

Experiment	$x - Range$	$Q^2$	$\Gamma_1^{NS}$	This Analysis
COMPASS	$0.004 < x < 0.7$	3	$0.175 \pm 0.009 \pm 0.015$	0.1421
HERMES	$0.021 < x < 0.9$	5	$0.1479 \pm 0.0169$	0.1381
E143	$0.03 < x < 0.8$	2	$0.149 \pm 0.016$	0.1276
E143	$0.03 < x < 0.8$	3	$0.164 \pm 0.023$	0.1301
E143	$0.03 < x < 0.8$	5	$0.141 \pm 0.013$	0.1327
E154	$0.03 < x < 0.8$	5	$0.168 \pm 0.010$	0.1327
E155	$0.03 < x < 0.8$	5	$0.176 \pm 0.008$	0.1327
SMC	$0 < x < 1$	10	$0.198 \pm 0.023$	0.1626
SMC	$0 < x < 1$	5	$0.174 \pm 0.024 \pm 0.012$	0.1569
JLAB	$0.001 < x < 0.8$	0.592	$0.1027 \pm 0.0228 \pm 0.0052$	0.123
JLAB	$0.001 < x < 0.8$	0.707	$0.0945 \pm 0.0201 \pm 0.0151$	0.1298
JLAB	$0.001 < x < 0.8$	0.844	$0.1021 \pm 0.0193 \pm 0.0174$	0.134
JLAB	$0.001 < x < 0.8$	1.20	$0.1307 \pm 0.0192 \pm 0.0145$	0.1323
JLAB	$0.001 < x < 0.8$	1.44	$0.1522 \pm 0.0186 \pm 0.0089$	0.1433
JLAB	$0.001 < x < 0.8$	1.71	$0.1605 \pm 0.0182 \pm 0.0069$	0.1318
JLAB	$0.001 < x < 0.8$	2.05	$0.1678 \pm 0.0177 \pm 0.0056$	0.1475
JLAB	$0.001 < x < 0.8$	2.44	$0.1666 \pm 0.0167 \pm 0.0045$	0.1492
JLAB	$0.001 < x < 0.8$	2.92	$0.1789 \pm 0.0106 \pm 0.0035$	0.1511
NN Collaboration	$0.021 < x < 0.9$	5	$0.1315 \pm 0.0144$	0.1381

TABLE II: Comparison of the integral over different  $x$  range at different scale of  $Q^2$ , as determined from the valon model, with the experimental results from COMPASS, HERMES, E143, E154, E155, SMC and JLAB also with the results from NN Collaborations.

It is interesting to investigate this issue in the polarized case.

The Regge pole model gives the following small  $x$  behavior of the structure functions  $g_1^i(x, Q^2)$ [28]

$$g_1^i(x, Q^2) = \gamma_i(Q^2)x^{-\alpha_i} \quad (14)$$

where  $g_1^i(x, Q^2)$  denote either singlet ( $g_1^s(x, Q^2) = g_1^p(x, Q^2) + g_1^n(x, Q^2)$ ) or non-singlet ( $g_1^{ns}(x, Q^2) = g_1^p(x, Q^2) - g_1^n(x, Q^2)$ ) combination of structure functions. Therefore we expect

that describe the small  $x$  behavior of  $g_1^{NS}$  and  $g_1^p$  structure functions with one and two Regge exponents respectively. It appears that the present  $g_1^{NS}$  data for available small  $x$  in the interval  $0.0001 < x < 0.01$  can be described with a single Regge type exchange as :

$$g_1^{NS} \equiv g_1^p - g_1^n \simeq Ax^{\alpha_{Regge}} \quad (15)$$

The Regge intercept which governs the small  $x$  physics has smooth  $Q^2$  dependence, in which case one would see  $g_1^{NS}$  rising like  $x^\alpha$  where  $-0.5 \leq \alpha \leq 0$  also at low  $Q^2$  and in the measured "small  $x$ " region [30]. This value varies from -0.13 to -0.3 when  $Q^2$  is moved from  $Q^2 = 2GeV^2$  to  $Q^2 = 10GeV^2$  in the valon model. According to the results of Ref.[31], asymptotic scaling of  $g_1^{NS}$  depends on one variable  $Q^2/x^2$  only, instead of two variables  $x$  and  $Q^2$  with the constant intercept equal to 0.42:

$$g_1^{NS} \simeq (Q^2/x^2)^{\Delta_{NS}/2} \simeq Q^{\Delta_{NS}} x^{-\Delta_{NS}}; \Delta_{NS} = 0.42 \quad (16)$$

However, it is valid for a very small  $x$  only. The applicability region of that analysis is  $x \leq 10^{-6}$ . Figure 6 shows the non-singlet spin structure function at  $Q^2 = 5GeV^2$  for small  $x$  region. In Figure 7, we fit the non-singlet spin structure function at small  $x$  ( $0.0001 < x < 0.01$ ) and find the associated  $\alpha_{Regge}$  :

$$g_1^{NS} = Ax^{\alpha_{Regge}}; A = 0.173, \alpha_{Regge} = -0.323 \quad (17)$$

### A. Scaling behavior of $g_1^p$ at small $x$

Having analyzed the small  $x$  behavior of  $g_1^{NS}$ , it is interesting to see how  $g_1^p$  behaves as  $x \rightarrow 0$ . Because the valon model has very good agreement with existent small  $x$  data, it is a good candidate to show the small  $x$  behavior of  $g_1^p$  at small  $x$ . Actually the results for this scaling behavior should be compared with small  $x$  data for  $g_1^p$ .

We show that the polarized proton structure function has this scaling behavior for  $1.2 < Q^2(GeV^2) < 100$  at small  $x$  ( $10^{-5} < x < 10^{-2}$ )

$$g_1^p(x, Q^2) = \sum_{i=1}^2 a_i f_i(Q^2) x^{\varepsilon_i} \quad (18)$$

Where  $a_i$  and  $\varepsilon_i$  are constants and the functions  $f_i(Q^2)$  have this simple general form

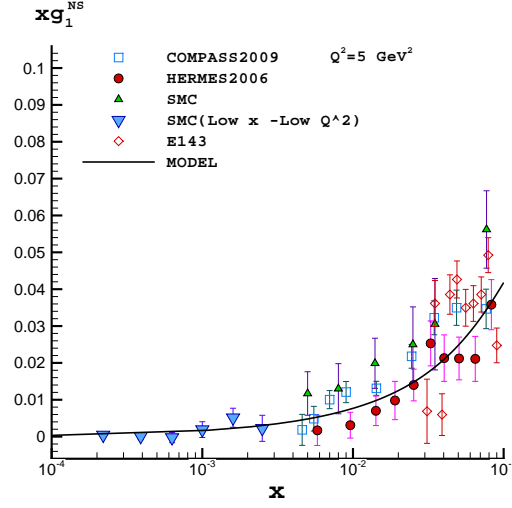


FIG. 6: (Color online) Small  $x$  behavior of  $xg_1^{NS}$  at  $Q^2 = 5\text{GeV}^2$  calculated by using the valon model in comparison with the experimental data .

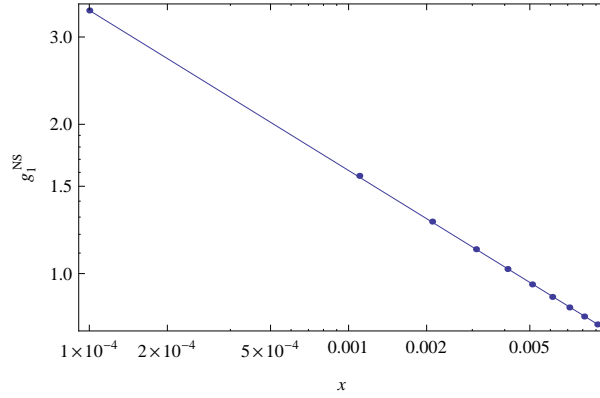


FIG. 7: Small  $x$  behavior of  $g_1^{NS}$  at  $Q^2 = 5\text{GeV}^2$  calculated by using the valon model and using the best fit to calculate the Regge exponent. Data points are from the model and the goodness of fit is:  $\chi^2 = 0.99$ .

$$f_2(Q^2) = \left(\frac{Q^4}{1 + Q_0^4}\right)^{D_i} \quad (19)$$

$$f_1(Q^2) = f_2(Q^2)g(Q^2) \quad (20)$$

$$(21)$$

Where

$$g(Q^2) = g_0 + g_1 \text{Log}(Q^2) + g_2 \text{Log}(Q^2)^2 + g_3 \text{Log}(Q^2)^3 \quad (22)$$

The results for parametrization of  $g_1^p$  are summarized in table 3. (We should noted that if we try to do a fit with only one regge exponent and only with the first term in Eq. 18, we give a fit with  $\chi^2 = 0.99$ . So, It is clear that we can describe the polarized proton structure function with two exponents,  $\chi^2 = 0.996$ , better than one exponent )

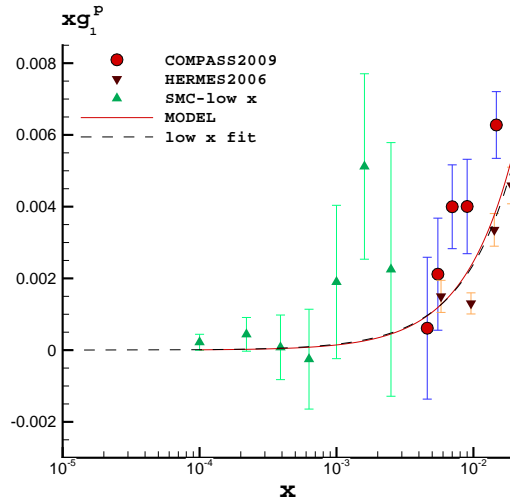


FIG. 8: (Color online) Small  $x$  behavior of  $xg_1^p$  at  $Q^2 = 5\text{GeV}^2$  calculated by using the valon model and using the best fit to calculate the scaling exponents at small  $x$ . The experimental data selected for a various of  $Q^2$  at small  $x$  region.

In Figure 8 we show the Small  $x$  behavior of  $xg_1^p$  at  $Q^2 = 5\text{GeV}^2$  calculated by using the valon model and using the best fit to calculate the scaling exponents,  $\varepsilon_i$ .

As a result, we concluded that only by using two scaling exponents, we can describe the small  $x$  behavior of  $g_1^p(x, Q^2)$  well. This behavior has also been seen in the small  $x$  tail of  $F_2^p(x, Q^2)$  structure function [28]. The question has been raised [32, 33] whether the observed rise on  $g_1^{NS}(x, Q^2)$  follows from one or two pomeron exchange( a polarized analogue of the one or two pomerons question). However, looking at  $g_1^p(x, Q^2)$ , it indeed requires two pomerons!

<i>Parameters</i>	<i>values</i>
$\varepsilon_1$	0.196
$\varepsilon_2$	0.094
$a_1$	0.0215
$a_2$	0.0513
$D_1$	0.759
$D_2$	0.434
$g_0$	17.538
$g_1$	-11.809
$g_2$	2.652
$g_3$	-0.200
$Q_0$	1.300
$\chi^2(\text{Goodness of fit})$	0.996

TABLE III: global fit parameters obtained by fitting the Eq. (4.4) with the data extracted from the valon model at small  $x$ .

## V. CONCLUSION

In this paper we calculated the non-singlet spin structure function,  $g_1^{NS}$ , of the nucleon in the valon model. The results of these calculations are in excellent agreement with all experimental data for the entire measured range of  $x$ . We also study the small  $x$  behavior of non-singlet spin structure function and the Regge behavior of  $g_1^{NS}$  to calculate the Regge exponent,  $\alpha_{Regge}$ . Finally, we studied the scaling behavior of  $g_1^p$  at small  $x$ . We conclude that only by using two scaling exponents, we can describe the small  $x$  behavior of  $g_1^p$  well. This is very similar to existence of two soft and hard pomeron to describe the small  $x$  behavior of  $F_2(x, Q^2)$  in unpolarized case. It is shown that the valon model can predict the polarized nucleon structure functions for the entire measured  $x$  range very well. The validity range for using the valon model is  $0.5 < Q^2(\text{GeV}^2) < 10^7$  and  $10^{-6} < x < 1$ . In this model sea quarks polarization is consistent with zero (the finding that was confirmed by very recent experiments at HERMES and COMPASS), so the polarized hadron structure functions can build only by polarized valence and gluon distribution by finding only one type of polarized

valon distribution for each kind of valons. Between  $0.3 < Q^2(\text{GeV}^2) < 0.5$  we should consider other effects such as combination of resonance physics and vector-meson dominance that are important at low  $Q^2$ . Chiral perturbation theory may describe the behavior of polarized nucleon structure close to threshold. These issues will be considered in our future works to improve our model.

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